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E: ISSN NO.: 2349-980X **Shrinkhla Ek Shodhparak Vaicharik Patrika**

A Two Unit Cold Standby System with Two Types of Repairman and Rest Period of Skilled Repairman

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Abstract

The paper deals with a two unit cold standby system with two phase repair by two repairman- (ordinary and skilled) and rest period of skilled repairman. The system comprises of two identical units and each unit of the system has two modes: normal (N) and total failure (F). As soon as an unit fails, it is first attended immediately by ordinary repairman and then the unit is attended by skilled repairman. The skilled repairman needs rest after working for a random period of time and after taking complete rest he again starts the repair of failed unit and the time already spent in its repair by skilled repairman goes to waste.

Keywords: Regenerative point, Reliability, Mean Time To System Failure, Busy period of repairman, Net Expected Profit.

Introduction

Two unit priority/ non-priority standby system models have been widely studied in the literature of reliability due to their frequent and significant use in modern business and industry. Various authors including Chandrashekhar ,Goyal and Kumar [8,1,9] and Srinivasan and Gupta[10,3] have studied two-unit standby systems by using different concepts such as imperfect switching device and two types of operation and repair, slow switching device, preventive maintenance. The common assumption taken in analyzing all above system models is that a single repairman repairs the failed unit continuously till the unit is repaired (as good as new). But realistic situations may arise when it is not possible for a repairman to repair a failed unit continuously for a long period of time due to his tiredness so that after a period of time his working efficiency may reduce.

Keeping the above fact in view, we analyze a two identical unit cold standby system model assuming that a failed unit first attended by ordinary repairman and then shifted to skilled repairman to make the failed unit as good as new i.e. the repair of a failed unit is completed in two phases and for each phase repair a separate repairman is required.

Using regenerative point technique, the following economic measures of the system effectiveness are obtained:

1. Reliability and Mean Time To System Failure (MTSF).
2. Point-wise and steady-state availabilities of the system and expected up time of the system during time (0, t).
3. The expected busy period of repairman in time interval (0,t).
4. Net expected profit earned by the system during time (0, t) and in steady- state.

Aim of the paper

The present paper explain the concept of a two identical unit cold standby system model assuming that a failed unit first attended by ordinary repairman and then shifted to skilled repairman to make the failed unit as good as new i.e. the repair of a failed unit is completed in two phases and for each phase repair a separate repairman is required. The skilled repairman needs rest after working for a random period of time and after taking complete rest he again starts the repair of failed unit and the time already spent in the repair by skilled repairman goes to waste.

Model Description and assumptions

1. The system comprises of two identical units. Initially one unit is operative and other is kept into cold standby.
2. Each unit of the system has two modes: normal (N) and total failure (F).

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3. The repair of a failed unit is completed in two different phases and for each phase repair a separate repairman is considered. These two repairman are named as ordinary and skilled repairman.
4. As soon as an unit fails, it is attended immediately by ordinary repairman and then the unit is attended by skilled repairman. Both repairman are always available with the system.
5. The skilled repairman needs rest after working for a random period of time and after taking complete rest he again starts the repair of failed unit and the time already spent in its repair by skilled repairman goes to waste.
6. The failure time distribution of an operating unit, repair time distribution of ordinary repairman and working time distribution of skilled repairman are taken as exponential with different parameters whereas repair time distribution and rest time distribution of skilled repairman are taken as general.
7. Each repaired unit by skilled repairman works as good as new.

Review of Literature

This paper based on a stochastic behavior of a two- identical unit cold standby system model assuming two modes - normal and total failure of the units. The repair of a failed unit is completed in two different phases and for each phase repair a seperate repairman is considered. These two repairman are named as ordinary and skilled repairman.The repair discipline is FCFS in respect of above jobs.

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Notations and states of the system

(a) Notations

λ : failure rate of a unit.

μ : repair rate by ordinary repairman.

G(.): cdf of repair time of a skilled repairman.

θ : rate of going a skilled repairman for rest.

H (.): cdf of rest time of the skilled repairman.

(b) Symbols used for the states of the system

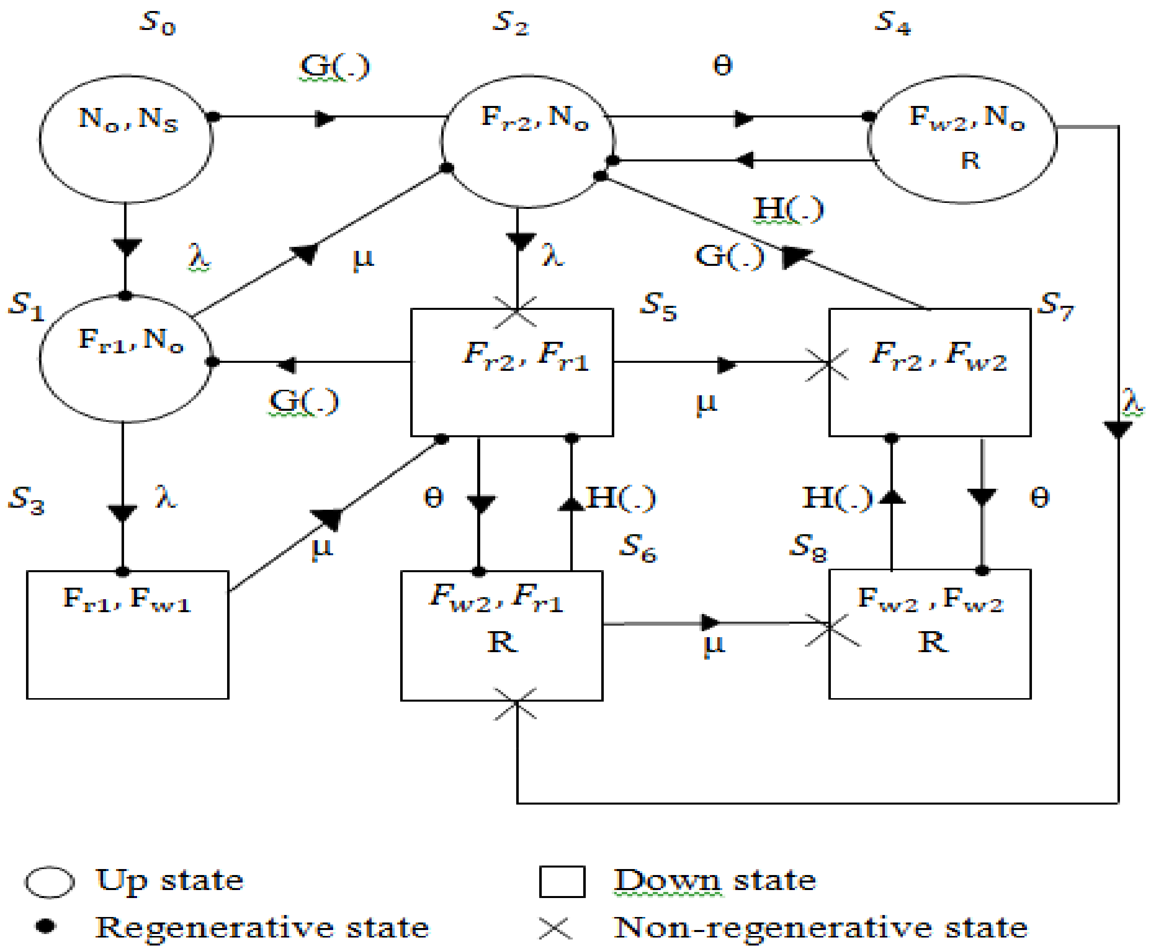
N_o, N_S : Unit in (N) mode and operative/standby.

F_{r1}, F_{r2} Unit in F mode and under repair by ordinary/skilled repairman.

F_{w1}, F_{w2} : Unit in F mode and waiting for ordinary/skilled repairman.

R : Skilled repairman is under rest.

TRANSITION DIAGRAM



The transition diagram of the system model is shown in the figure where we observe that the epochs of entrance from S_2 to S_5, S_4 to S_6, S_5 to S_7 and from S_6toS_8 are non-regenerative whereas all other entrance points are regenerative.

Transition Probabilities and Sojourn Times

(a) The steady- state unconditional and conditional transition probabilities can be obtained as follows

$p_{01} = 1$

$p_{12} = \frac{\mu}{\mu+\lambda}$

$p_{13} = \frac{\lambda}{\mu+\lambda}$

$p_{20} = \widetilde{G}\ (\theta+\lambda)$

$p_{24} = 1 - \widetilde{G}(\theta+\lambda)$

$p_{21}^{(5)} = \frac{\lambda}{\lambda-\mu}[\widetilde{G}(\theta+\mu) - \widetilde{G}(\theta+\lambda)]$

$p_{26}^{(5)} = \frac{\lambda}{\lambda-\mu}[\widetilde{G}(\theta+\lambda) - \widetilde{G}(\theta+\mu)]$

$p_{22}^{(5,7)} = \frac{\lambda}{\lambda-\mu}[\widetilde{G}(\theta) - \widetilde{G}(\theta+\mu)] - \frac{\mu}{\lambda-\mu}[\widetilde{G}(\theta) - \widetilde{G}(\lambda+\theta)]$

$p_{28}^{(5,7)} = \frac{\lambda}{\lambda-\mu}[\widetilde{G}(\theta+\mu) - \widetilde{G}(\theta)] - \frac{\mu}{\lambda-\mu}[\widetilde{G}(\lambda+\theta) - \widetilde{G}(\theta)]$

$p_{35} = 1$

$p_{42} = \widetilde{H}\ (\lambda)$

$p_{45}^{(6)} = \frac{\lambda}{\lambda-\mu}[\widetilde{H}(\mu) - \widetilde{H}(\lambda)]$

$p_{47}^{(6,8)} = \frac{\lambda}{\lambda-\mu}[1 - \widetilde{H}(\mu)] - \frac{\mu}{\lambda-\mu}[1 - \widetilde{H}(\lambda)]$

$p_{51} = \widetilde{G}\ (\theta+\mu)$

$p_{56} = 1 - \widetilde{G}\ (\theta+\mu)$

$p_{52}^{(7)} = \widetilde{G}(\theta) - \widetilde{G}(\theta+\mu)$

$p_{58}^{(7)} = \widetilde{G}(\mu+\theta) - \widetilde{G}(\theta)]$

$p_{65} = \widetilde{H}\ (\mu)$

$p_{67}^{(8)} = 1 - \widetilde{H}(\mu)$

$p_{72} = \widetilde{G}\ (\theta)$

$p_{78} = 1 - \widetilde{G}\ (\theta)$

$p_{87} = 1$

It can easily verified that

$p_{12} + p_{13} = 1$

$p_{20} + p_{24} + p_{21}^{(5)} + p_{26}^{(5)} + p_{22}^{(5,7)} + p_{28}^{(5,7)} = 1$

$p_{42} + p_{45}^{(6)} + p_{47}^{(6,8)} = 1$

$p_{51} + p_{56} + p_{52}^{(7)} + p_{58}^{(7)} = 1$

$p_{65} + p_{67}^{(8)} = 1$

$p_{72} + p_{78} = 1$

(b) The mean sojourn time in various states as follows

$\Psi_0 = \int P(T_0>t)\ dt = 1/\lambda$

$\Psi_1 = 1/\mu + \lambda$

$\Psi_2 = \frac{1 - \widetilde{G}\ (\theta+\lambda)}{\theta+\lambda}$

$\Psi_3 = 1/\mu$

$\Psi_4 = \frac{1 - \widetilde{H}(\ \lambda)}{\lambda}$

$\Psi_5 = \frac{1 - \widetilde{G}\ (\mu+\theta)}{\mu+\theta}$

$\Psi_6 = \frac{1 - \widetilde{H}(\ \mu)}{\mu}$

$\Psi_7 = \frac{1 - \widetilde{G}(\ \theta)}{\theta}$

$\Psi_8 = \int \overline{H}\ (t)\ dt$

Analysis of Characteristics

(a)Reliability and MTSF

To determine $R_i(t)$, the reliability of the system when system initially starts from regenerative state S_i, we assume that failed states S_3 ,S_5 ,S_6 ,S_7 and S_8 of the system as absorbing. Using simple probabilistic arguments in regenerative point technique, the value of $R_0(t)$ in terms of its Laplace transform is

$$R_0^*(s) = \frac{(1-q_{24}^*q_{42}^*)Z_0^* + q_{01}^*(1-q_{24}^*q_{42}^*)Z_1^* + q_{01}^*q_{12}^*Z_2^* + q_{01}^*q_{12}^*q_{24}^*Z_4^*}{1-q_{24}^*q_{42}^* - q_{01}^*q_{12}^*q_{20}^*}$$

where $Z_0^*, Z_1^*, Z_2^*$and Z_4^* are the L.T. of

$Z_0(t) = e^{-\lambda t}$, $Z_1(t) = e^{-(\lambda+\mu)t}$

$Z_2(t) = e^{-(\theta+\lambda)t}\overline{G}\ (t)$ $Z_4(t) = e^{-\lambda t}\overline{H}(t)$

The mean time to system failure is given by

$E\ (T_0) = \lim_{s\to 0} R_0^*(s)$

$$E(T_0) = \frac{[1-p_{24}p_{42}](\Psi_0+\Psi_1)+p_{12}\Psi_2+p_{12}p_{24}\Psi_4}{1-p_{24}p_{42}-p_{12}p_{20}}$$

(b) Availability Analysis

Let $A_i(t)$be the probability that the system is up at epoch t, when initially system starts operation from state $S_i\varepsilon E$. Using the regenerative point technique and the tools of Laplace transforms, one can obtain the value of $A_0(t)$ in terms of their Laplace Transformations i.e. $A_0^*(s)$ .Then the steady-state availability of the system is given by

$$A_0 = N_1/D_1$$

where,

$N_1 = U_0\Psi_0 + U_1\Psi_1 + U_2\Psi_2 + U_4\Psi_4$

The values of U_0, U_1, U_2 and U_4 can be obtained from

$N_1(0) =$

$$\begin{vmatrix} \Psi_0 & -p_{01} & 0 & 0 & 0 & 0 & 0 \\ \Psi_1 & 1 & -p_{12} & 0 & -p_{13}p_{35} & 0 & 0 \\ \Psi_2 & -p_{21}^{(5)} & 1\text{-}p_{22}^{(5,7)} & -p_{24} & 0 & -p_{26}^{(5)} & -p_{28}^{(5,7)}p_{87} \\ \Psi_4 & 0 & -p_{42} & 1 & -p_{45}^{(6)} & 0 & -p_{47}^{(6,8)} \\ 0 & -p_{51} & -p_{52}^{(7)} & 0 & 1 & -p_{56} & -p_{58}^{(7)}p_{87} \\ 0 & 0 & 0 & 0 & -p_{65} & 1 & -p_{67}^{(8)} \\ 0 & 0 & -p_{72} & 0 & 0 & 0 & 1\text{-}p_{78}p_{87} \end{vmatrix}$$

$$D_1 = U_0\Psi_0 + U_1\Psi_1 + U_2\Psi_2 + U_3\Psi_3 + U_4\Psi_4 + U_5\Psi_5 + U_6\Psi_6 + U_7\Psi_7 + U_8\Psi_8$$

The values of $U_0, U_1, U_2, U_3, U_4, U_5, U_6, U_7, U_8$ can be find from

$D_1(0) =$

$$\begin{vmatrix} 1 & -p_{01} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -p_{12} & 0 & -p_{13}p_{35} & 0 & 0 \\ -p_{20} & -p_{21}^{(5)} & 1\text{-}p_{22}^{(5,7)} & -p_{24} & 0 & -p_{26}^{(5)} & -p_{28}^{(5,7)}p_{87} \\ 0 & 0 & -p_{42} & 1 & -p_{45}^{(6)} & 0 & -p_{47}^{(6,8)} \\ 0 & -p_{51} & -p_{52}^{(7)} & 0 & 1 & -p_{56} & -p_{58}^{(7)}p_{87} \\ 0 & 0 & 0 & 0 & -p_{65} & 1 & -p_{67}^{(8)} \\ 0 & 0 & -p_{72} & 0 & 0 & 0 & 1\text{-}p_{78}p_{87} \end{vmatrix}$$

(c)Busy Period Analysis

(1) Due to ordinary repairman

Let $B_i^I(t)$ be the probability that the ordinary repairman is busy at epoch t, when initially system starts functioning from state $S_i\varepsilon E$. Using the

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regenerative point technique and the tools of Laplace transform, one can obtain the value of $B_i^I(t)$ in terms of their Laplace transform i.e.$B_i^{I*}(s)$. The property that the ordinary repairman is busy in long run is given by

$$B_0^I = N_2/D_1$$

Where,

$N_2 = K\Psi_1 + U_5\Psi_5 + U_6\Psi_6$

The values of U_5 and U_6 can be find from

$N_2(0) =$

$$\begin{vmatrix} 0 & -p_{01} & 0 & 0 & 0 & 0 & 0 \\ K & 1 & -p_{12} & 0 & -p_{13}p_{35} & 0 & 0 \\ 0 & -p_{21}^{(5)} & 1-p_{22}^{(5,7)} & -p_{24} & 0 & -p_{26}^{(5)} & -p_{28}^{(5,7)}p_{87} \\ 0 & 0 & -p_{42} & 1 & -p_{45}^{(6)} & 0 & -p_{47}^{(6,8)} \\ \Psi_5 & -p_{51} & -p_{52}^{(7)} & 0 & 1 & -p_{56} & -p_{58}^{(7)}p_{87} \\ \Psi_6 & 0 & 0 & 0 & -p_{65} & 1 & -p_{67}^{(8)} \\ 0 & 0 & -p_{72} & 0 & 0 & 0 & 1-p_{78}p_{87} \end{vmatrix}$$

The value of D_1 is same as given in (b).

(2) Due to skilled repairman-

Let $B_i^{II}(t)$ be the probability that the skilled repairman is busy at epoch t, when initially system starts functioning from state $S_i \varepsilon$ E. The property that the skilled repairman is busy in long run is given by

$$B_0^{II} = N_3/D_1$$

Where,

$N_3 = U_2\Psi_2 + U_5\Psi_5 + U_7\Psi_7$

The values of U_2, U_5 and U_7 can be find from

$N_3(0) =$

$$\begin{vmatrix} 0 & -p_{01} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -p_{12} & 0 & -p_{13}p_{35} & 0 & 0 \\ \Psi_2 & -p_{21}^{(5)} & 1-p_{22}^{(5,7)} & -p_{24} & 0 & -p_{26}^{(5)} & -p_{28}^{(5,7)}p_{87} \\ 0 & 0 & -p_{42} & 1 & -p_{45}^{(6)} & 0 & -p_{47}^{(6,8)} \\ \Psi_5 & -p_{51} & -p_{52}^{(7)} & 0 & 1 & -p_{56} & -p_{58}^{(7)}p_{87} \\ 0 & 0 & 0 & 0 & -p_{65} & 1 & -p_{67}^{(8)} \\ \Psi_7 & 0 & -p_{72} & 0 & 0 & 0 & 1-p_{78}p_{87} \end{vmatrix}$$

The value of D_1 is same as given in (b).

(d) Cost Benefit Analysis

Let K_0 is the per unit up time revenue by the system due to the operation of any unit and K_1 is the repair cost per unit of time when ordinary repairman busy in the repair and K_2 is the repair cost per unit of time when skilled repairman busy in the repair. Then the net expected profit incurred by the system during time interval (0, t) is given by

$$P(t) = K_0\mu_{up}(t) - K_1\mu_b^I(t) - K_2\mu_b^{II}(t)$$

The expected profit per- unit time in steady-state is

$$P = K_0A_0 - K_1B_0^I - K_2B_0^{II}$$

Conclusion

This paper explain the importance of introducing inspection policy to the system having different kind of repairman i.e. ordinary repairman and skilled repairman for obtaining the effectiveness of different reliability measures. Thus the results obtained, provides an effective information and new ideas for new researchers and companies to prefer such conditions for same system.

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